



# **Isotherm Testing: Procedures and Application of Results**

**Gary Amy**

Professor of Environmental Engineering  
University of Colorado USA



# Terminology

- (ad)sorption  $\Rightarrow$  accumulation  
@ solid/solvent interface
- (ad)sorbent  $\Rightarrow$  solid phase material
- (ad)sorbate  $\Rightarrow$  target compound(s)
  - ☐ arsenate (or arsenite) ✓
  - ☐ single vs. multi-sorbate
    - interferants
    - competition
- solvent  $\Rightarrow$  water
- knowledgebase  $\Rightarrow$  activated carbon + organics



# Adsorbent Forms and Properties

- Granular vs. Powdered
  - Fixed Bed vs. Slurry Reactor (or Membrane)
- Porous vs. Non-Porous
  - e.g., GFH vs. SMI
  - Kinetics
- (Specific) Surface Area ( $\text{m}^2/\text{g}$ )
- (Surface) Charge ( $\text{pH}_{\text{ZPC}}$ )
- Site Density ( $\#/\text{nm}^2$ )
- Mineralogy
  - Crystalline vs. Amorphous
  - Various Iron Oxides, etc. vs. Ion Exchange Resins
  - Strictly Adsorbent vs. Reactive media (e.g.,  $\text{MnO}_2$ )



# Influential Factors

## ■ Temperature

- Exo- versus Endothermic  $\Rightarrow$  *Isotherm*
- Exo  $\Rightarrow$  adsorption  $\propto 1/\text{temperature}$ ;  
Endo  $\Rightarrow$  adsorption  $\propto \text{temperature}$
- Adsorption
  - Generally exothermic
  - But higher diffusion at higher temperature often offsets higher temperature effects

## ■ pH

- $\text{H}_2\text{AsO}_4^-$  vs.  $\text{HAsO}_4^{2-}$

## ■ Interferants

- e.g., Phosphate



# Mechanisms of Arsenic Adsorption

- Physical Adsorption ✗
  - van der Waals Forces
- Chemisorption ✓
  - Surface Complexation
- Exchange Adsorption ✓
  - Ion Exchange



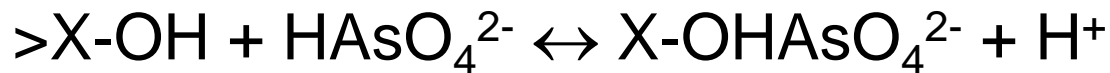
# Adsorption Mechanisms

## ■ Chemisorption:

- Ligand (L) Exchange



- Surface Complexation



## ■ Exchange Sorption:

- $>\text{X-Cl} + \text{L}^- \leftrightarrow >\text{X-L} + \text{Cl}^-$

## ■ Influential Factors

- pH:  $\text{H}_2\text{AsO}_4^-$ ,  $\text{HAsO}_4^{2-}$

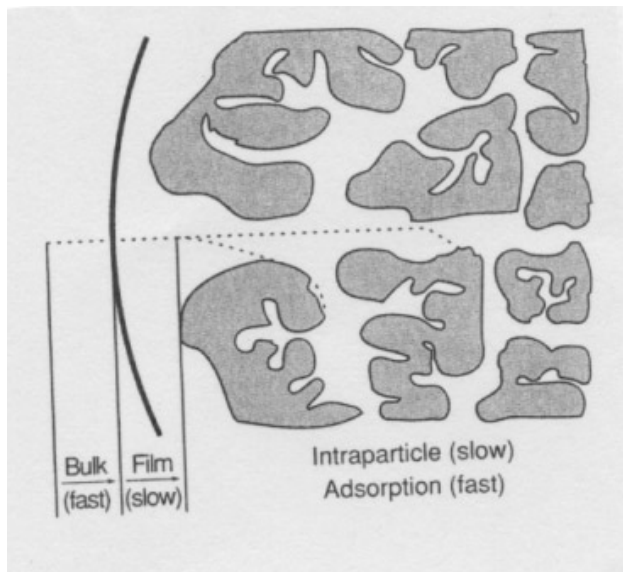
- $\text{pH}_{\text{ZPC}}$  of Adsorbent; if  $\text{pH} < \text{pH}_{\text{ZPC}}$ , “+”

- Inner (bond) vs. Outer Sphere Complexes (ion pair)

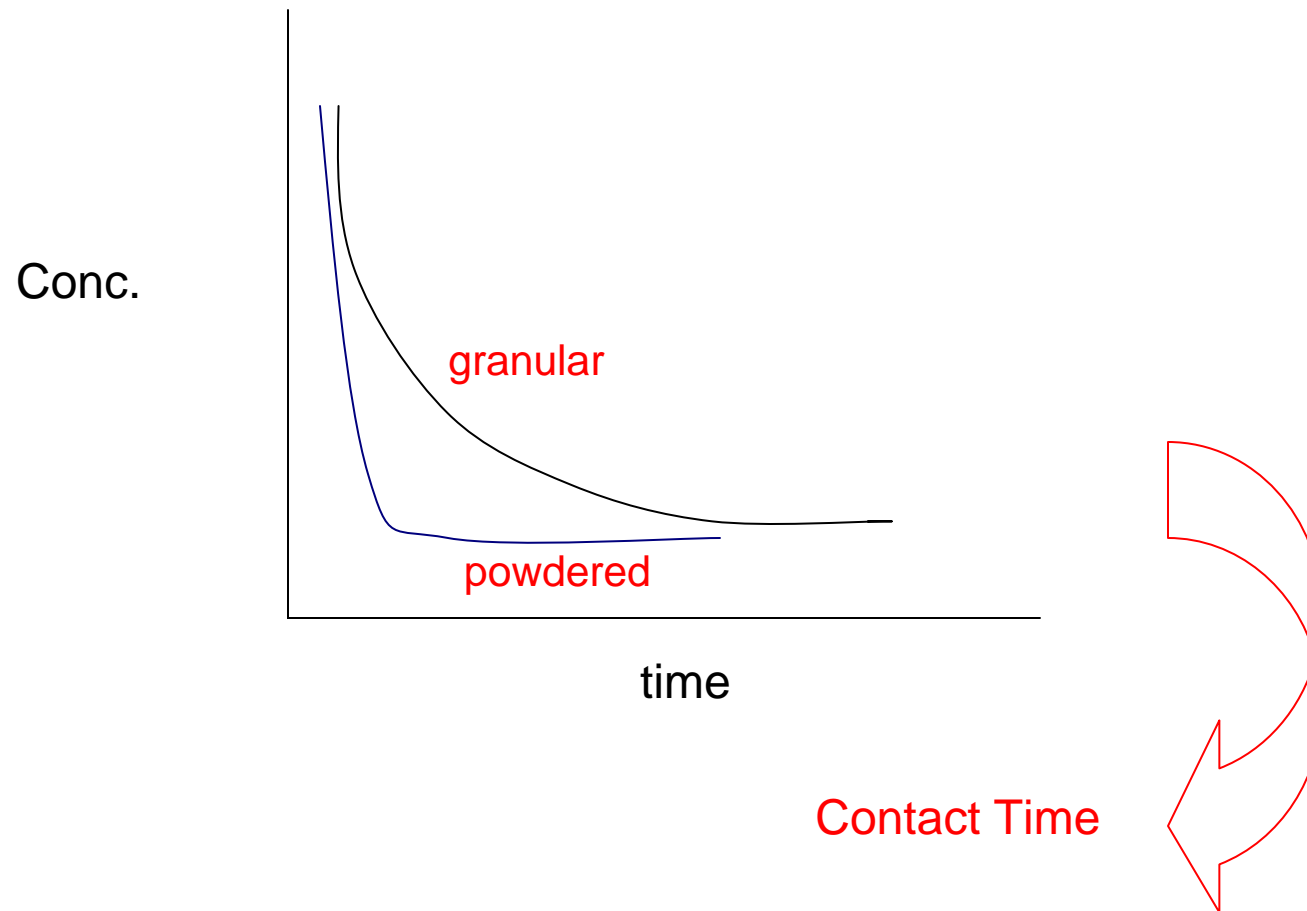
# Mass Transfer Steps

- (External) Film Diffusion
- (Internal) Pore and/or Surface Diffusion
  - Generally Rate-Limiting
- Surface Reaction

*Kinetics*

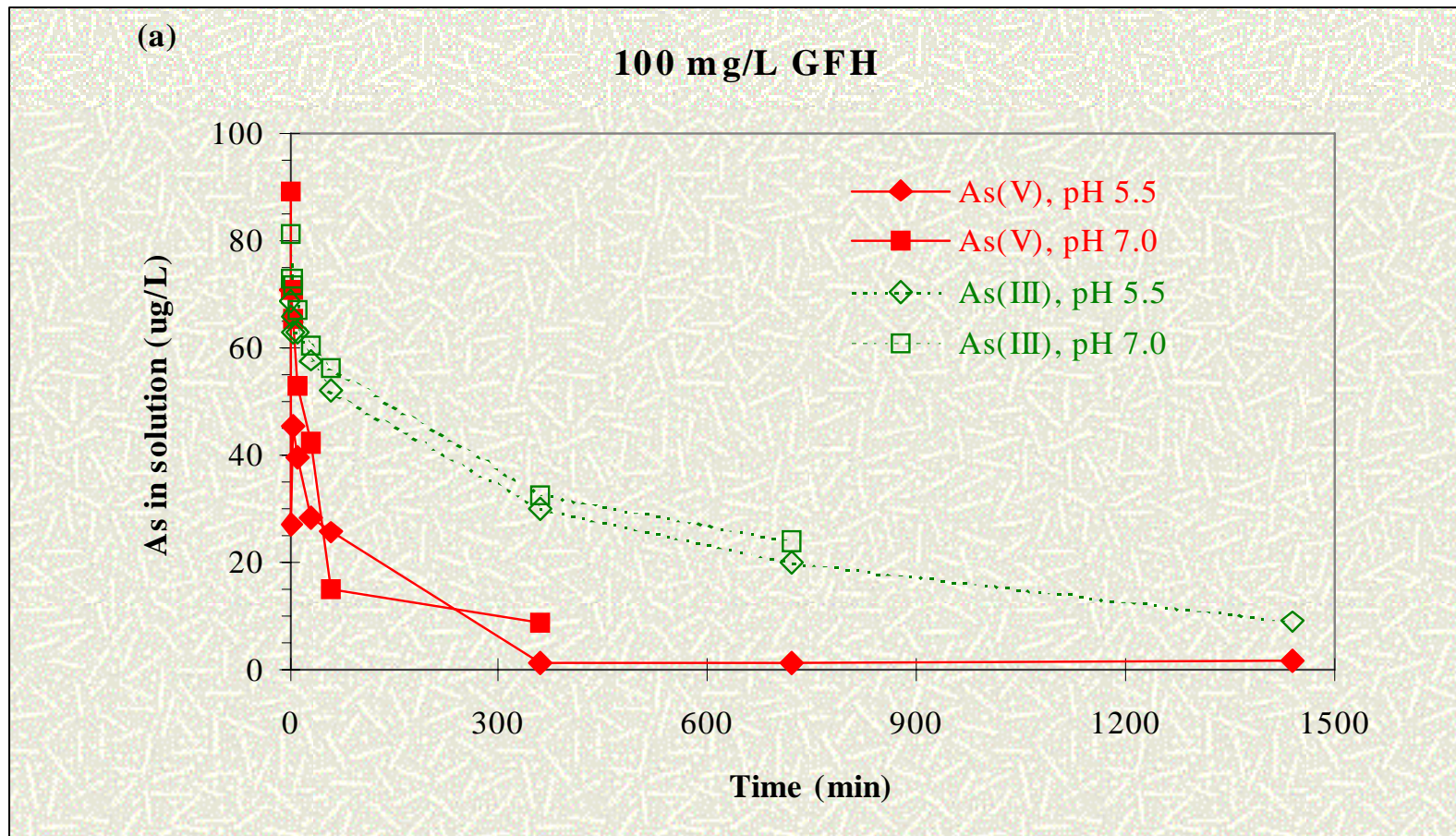


# Kinetics



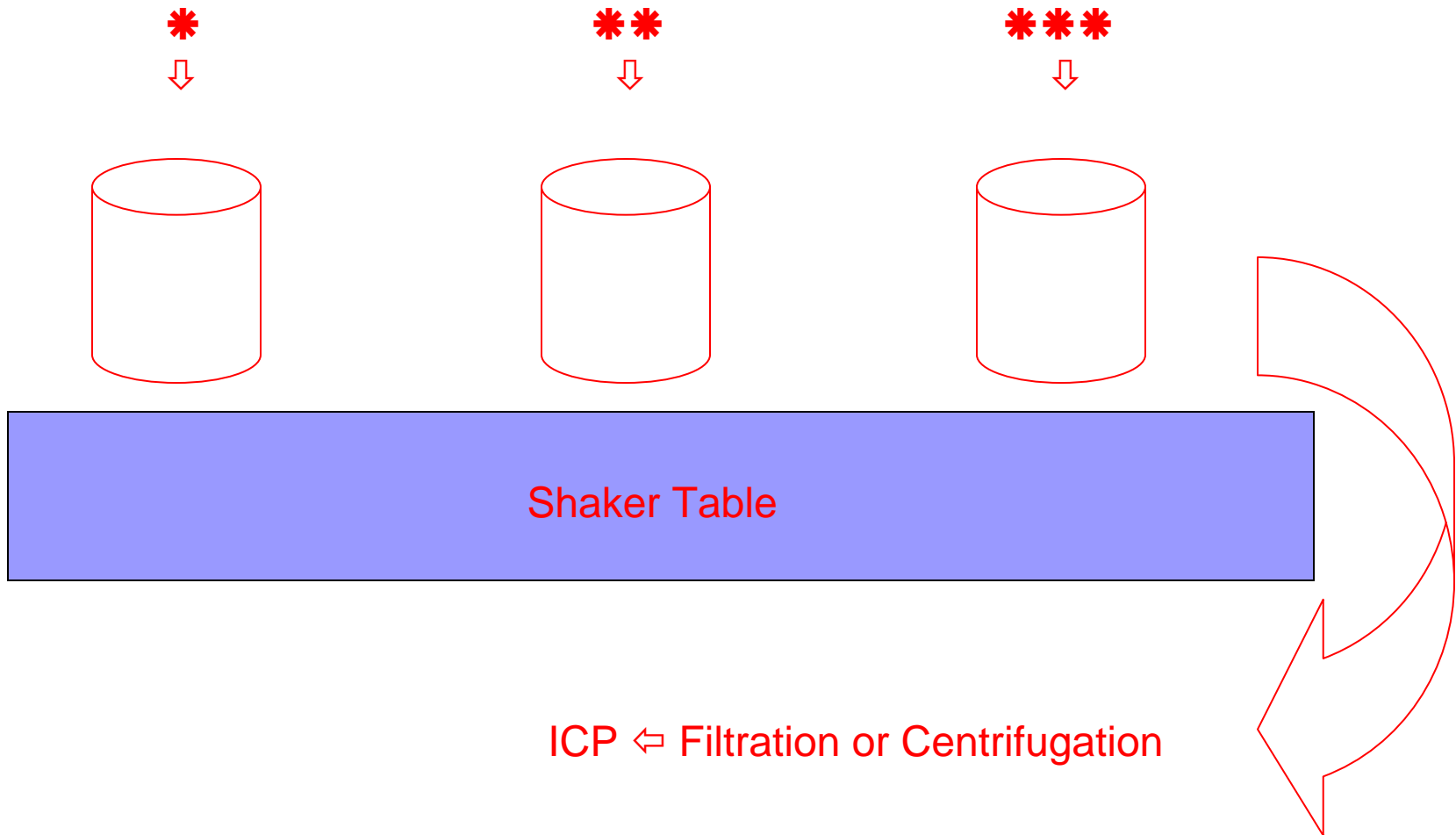


# Kinetics



Other Studies: up to 96 hours

# Bottle Point Isotherms





# Bottle Point Experiments

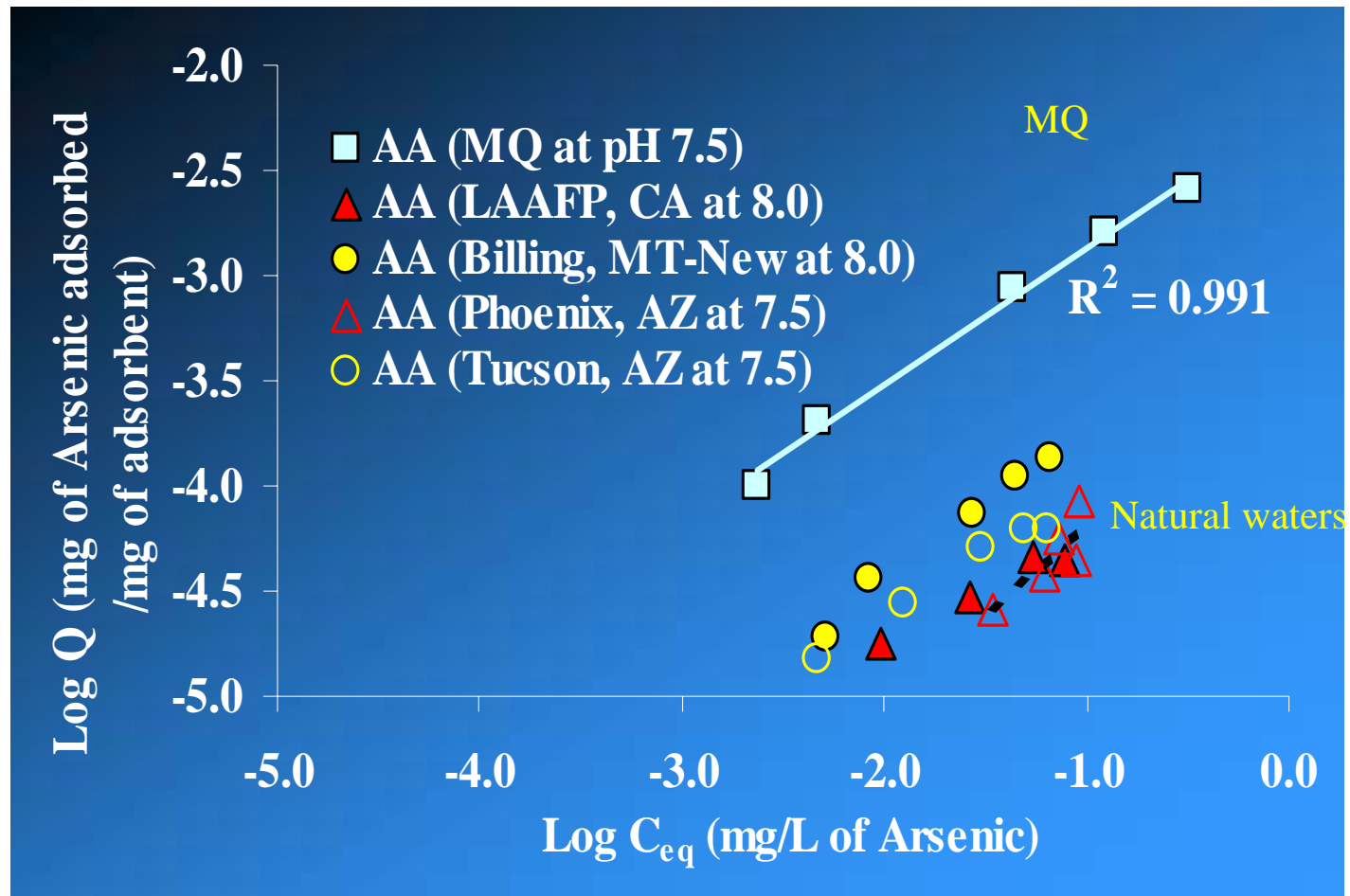
- Concentration Constant  $\Rightarrow$  Vary Adsorbent
  - natural water
- Adsorbent Constant  $\Rightarrow$  Vary Concentration
  - e.g., synthetic water
- Multi-Adsorbate
  - $C_0$  of arsenic and interferant affects isotherm



# (Equilibrium) Isotherms

- $q = x/m = (V(C_0 - C))/m$ 
  - $q \Rightarrow$  solid phase conc. (e.g., ug/mg)
  - $C \Rightarrow$  equil. water-phase conc. (e.g., ug/L)
    - Initial ( $C_0$ ) vs. Equilibrium ( $C$ )
  - $m \Rightarrow$  adsorbent concentration (e.g., mg/L)
  - $V \Rightarrow$  volume (e.g., L)
  - $x \Rightarrow$  mass adsorbed (e.g., ug)
- Batch Equilibrium Tests
  - Equilibration Time:
    - Powdered  $\Rightarrow$  minutes to hours
    - Granular  $\Rightarrow$  hours to days
    - Pulverize granular media?

# Activated Alumina Isotherms





# Isotherm Equations

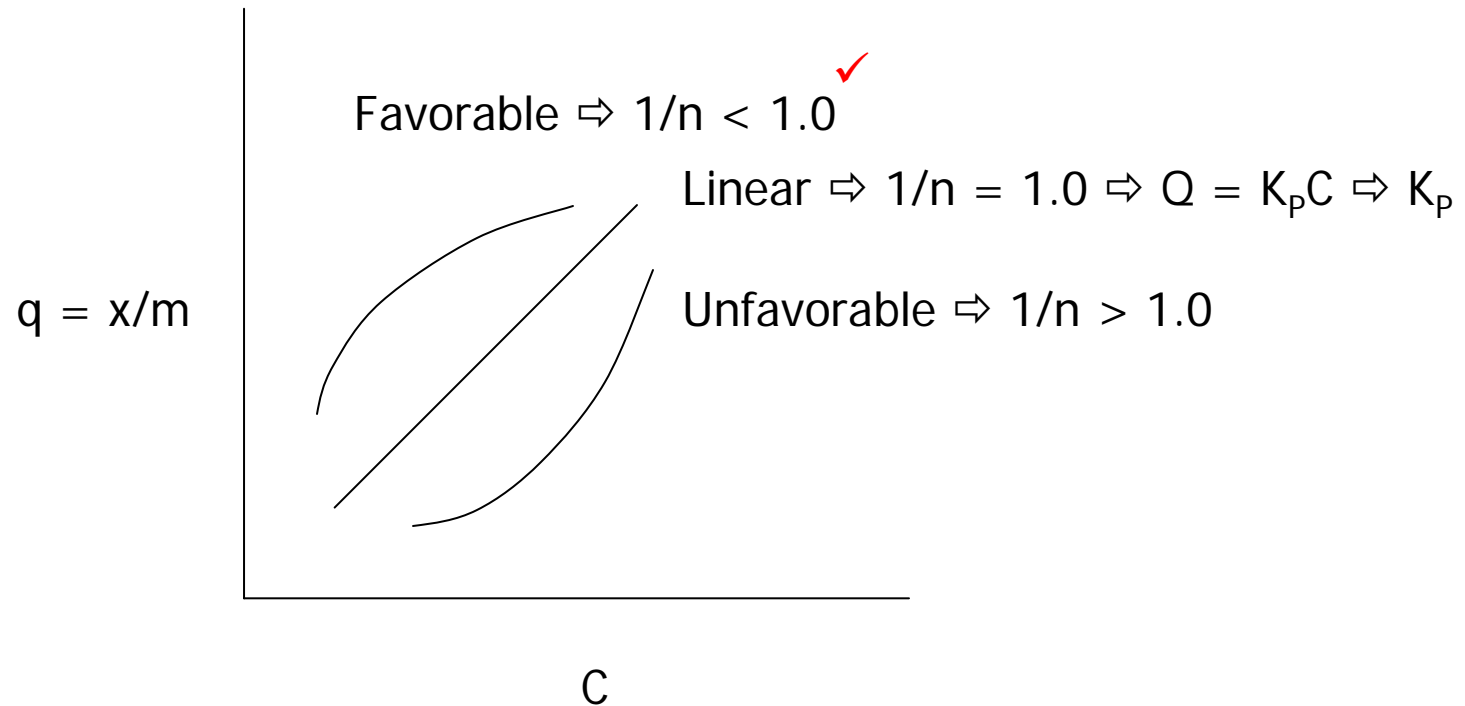
## ■ Freundlich

- Empirical; good data fit of intermediate range data
- $q = K_F C^{1/n}$  (or  $q = KC^n$  in Europe)
- $q \Rightarrow$  solid phase conc. (ug/mg, moles/g, etc.)
- $C \Rightarrow$  equil. water-phase conc. (ug/l, moles/L, etc.)
- $K_F$  &  $1/n \Rightarrow$  empirical constants
  - $K_F \Rightarrow$  capacity parameter (units!)
  - $1/n \Rightarrow$  index of „favorable“ vs. „unfavorable“ adsorption

## ■ Langmuir

- Theoretical; good fit of higher and lower range data
- $q = (Q_{\max} K_L C) / (1 + K_L C)$
- $Q_{\max} \Rightarrow$  max. surface conc.
  - indicative of „monolayer“ coverage or site saturation ✓
- $K_L \Rightarrow$  constant

# Favorable vs. Unfavorable Adsorption



## Determination of Isotherm Constants – cont.

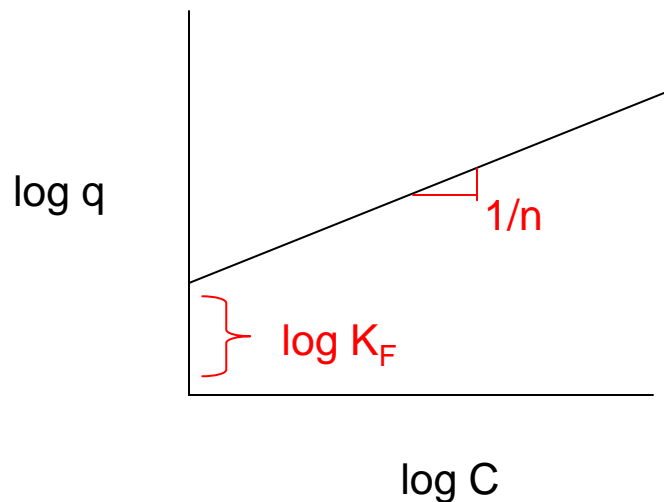
### ■ Linearization of Freundlich Equation

□  $\log q = \log K_F + 1/n \log C$

□ log-log plot

■ Intercept  $\Rightarrow K_F$

■ Slope  $\Rightarrow 1/n$







## Determination of Isotherm Constants – cont.

### ■ Linearization of Langmuir Equation

- $1/q = 1/Q_{\max} + (1/bQ_{\max})(1/C)$

- Plot of  $1/q$  vs.  $1/C$

- Intercept  $\Rightarrow 1/Q_{\max}$

- Slope  $\Rightarrow 1/K_L Q_{\max}$



# Adsorption Capacity

- $q$  vs.  $C$

- $q_{eq}$  vs.  $C_{eq}$

- $q_0$  vs.  $C_0$

- $q_{10}$  or  $q_{50}$  vs.  $C = 10$  or  $50$  ug/L

- mass based  $q$  (ug/mg)

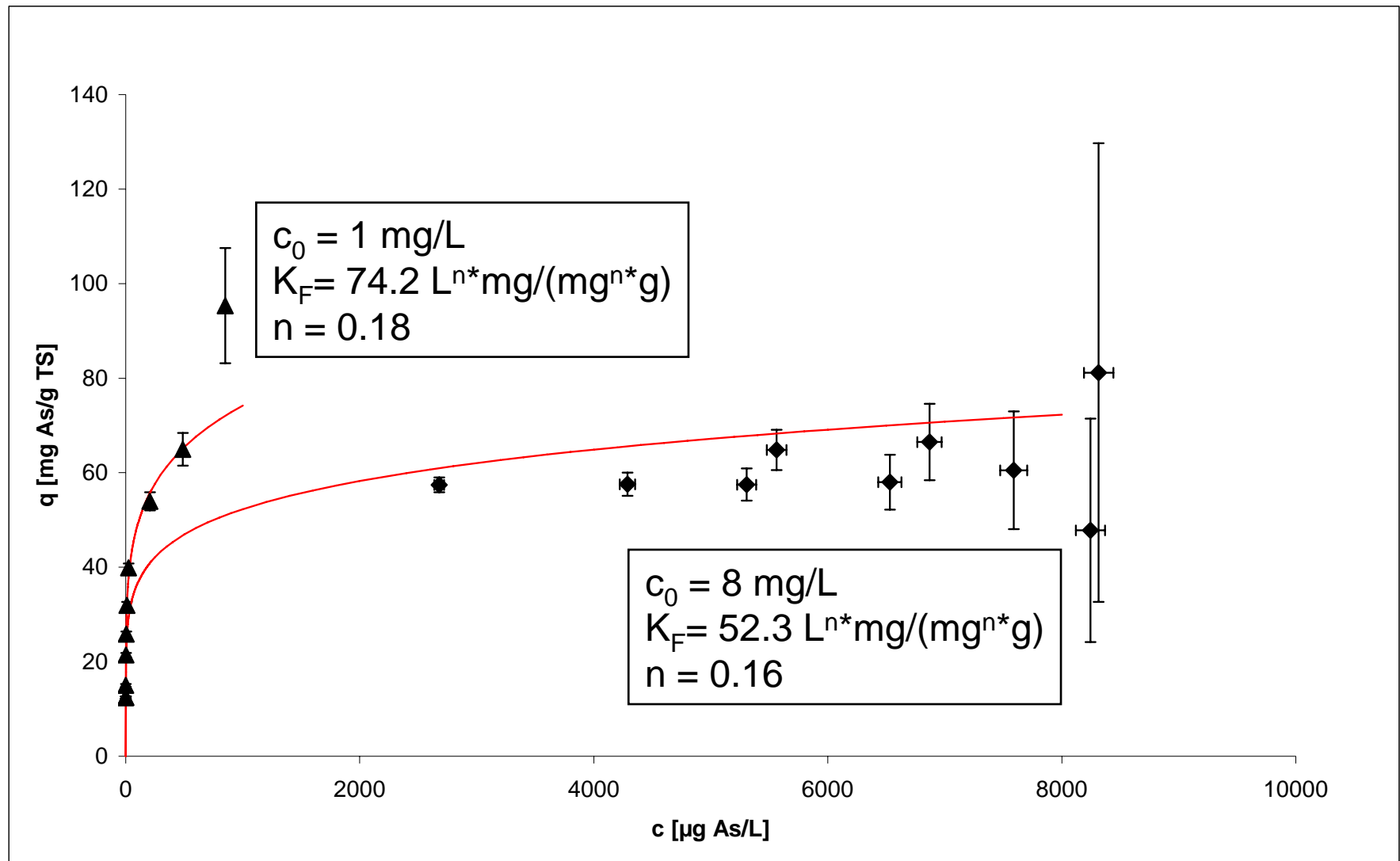
- vs. surface area based  $q$  (ug/cm<sup>2</sup>)



# *A Robust Isotherm*

- Lower Level Resolution
  - Adsorbate Limited
- Higher Level Resolution
  - Adsorbent Limited

## Isotherm: Higher and Lower Concentration Ranges





# Adsorption of Mixtures

- Multiple Adsorbates
- Reduced Single Adsorbate Capacity

# Competitive Langmuir Equation

- $q_i = (Q_{\max,i} K_{L,i} C_i) / (1 + \sum_{j=1}^n (K_{L,j} C_j))$ 
  - $n \Rightarrow$  number of (ad)sorbates
  - $C_i \Rightarrow$  equil. conc. of  $i$  sorbate in a  $j$  sorbate mixture
  - $q_i$  &  $K_{L,i} \Rightarrow$  single sorbate parameters
- For two (Ad)sorbates:
  - $q_1 = (Q_{\max,1} K_{L,1} C_1) / (1 + K_{L,1} C_1 + K_{L,2} C_2)$
  - $q_2 = (Q_{\max,2} K_{L,2} C_2) / (1 + K_{L,1} C_1 + K_{L,2} C_2)$
  - $Q_{\max,1}, Q_{\max,2}, K_{L,1}, K_{L,2} \Rightarrow$  derived from single sorbate experiments
  - $C_1, C_2 \Rightarrow$  derived from competitive sorbate experiments
  - $q_1, q_2 \Rightarrow$  calculated from competitive adsorption



## Information Provided by Isotherms

### ■ Powdered Adsorbent Applications

□ If  $m$  is specified,  $C = ?$

■  $q = x/m = (V(C_0 - C))/m = K_F C^{1/n}$

□ If  $C$  is specified,  $m = ?$

■  $q = x/m = (V(C_0 - C))/m = K_F C^{1/n}$

### ■ Granular Adsorbent Applications

□ Column-mode capacity  $\Rightarrow q_0$  (vs.  $C_0$ )



How does isotherm capacity  
relate to column capacity?

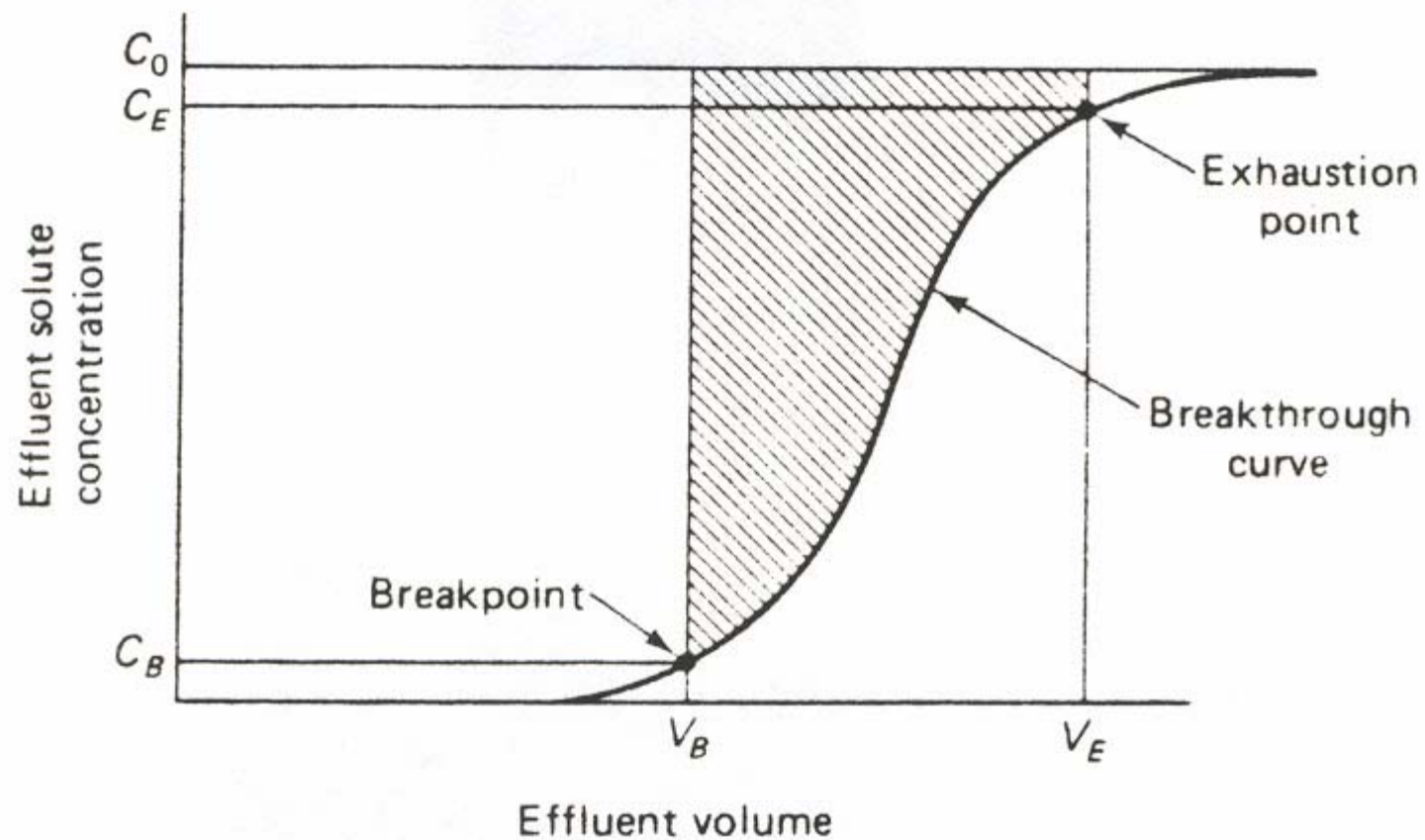




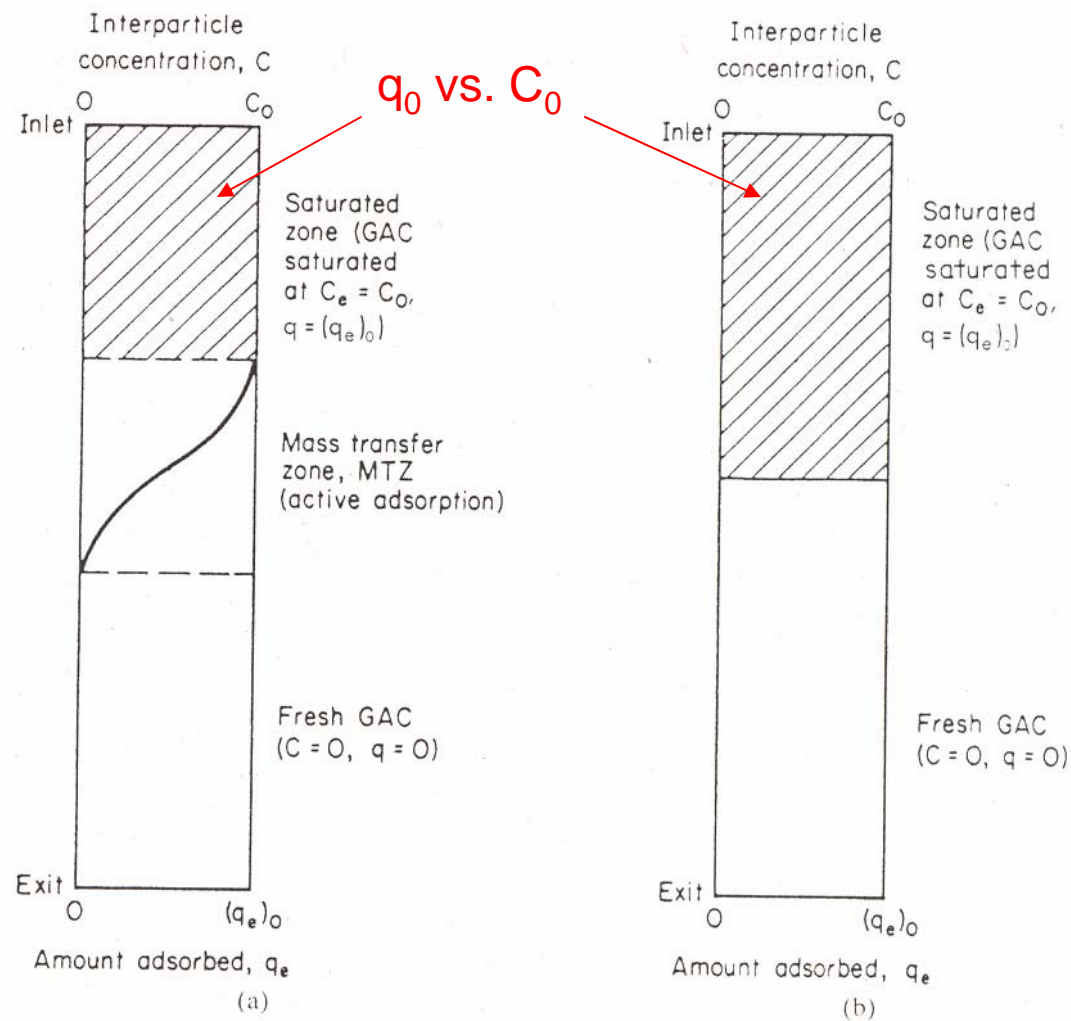
# GAC Column Performance

- **C**(oncentration) to Breakthrough or Exhaustion  
VS.  
**t**(ime) or  
**V**(olume),  $\Sigma V$ , or  
Bed V(olumes), **BVs** =  $\Sigma V / V_{\text{empty}}$

# Fixed Bed Column: Idealized Breakthrough Curve (BTC)



# Adsorption Column with (a) and without (b) MTZ



# Mass Adsorbed at Breakthrough or Exhaustion

- Integration of BTC:  $C$  (mg/L) vs.  $V$  (L)

$$V = V$$

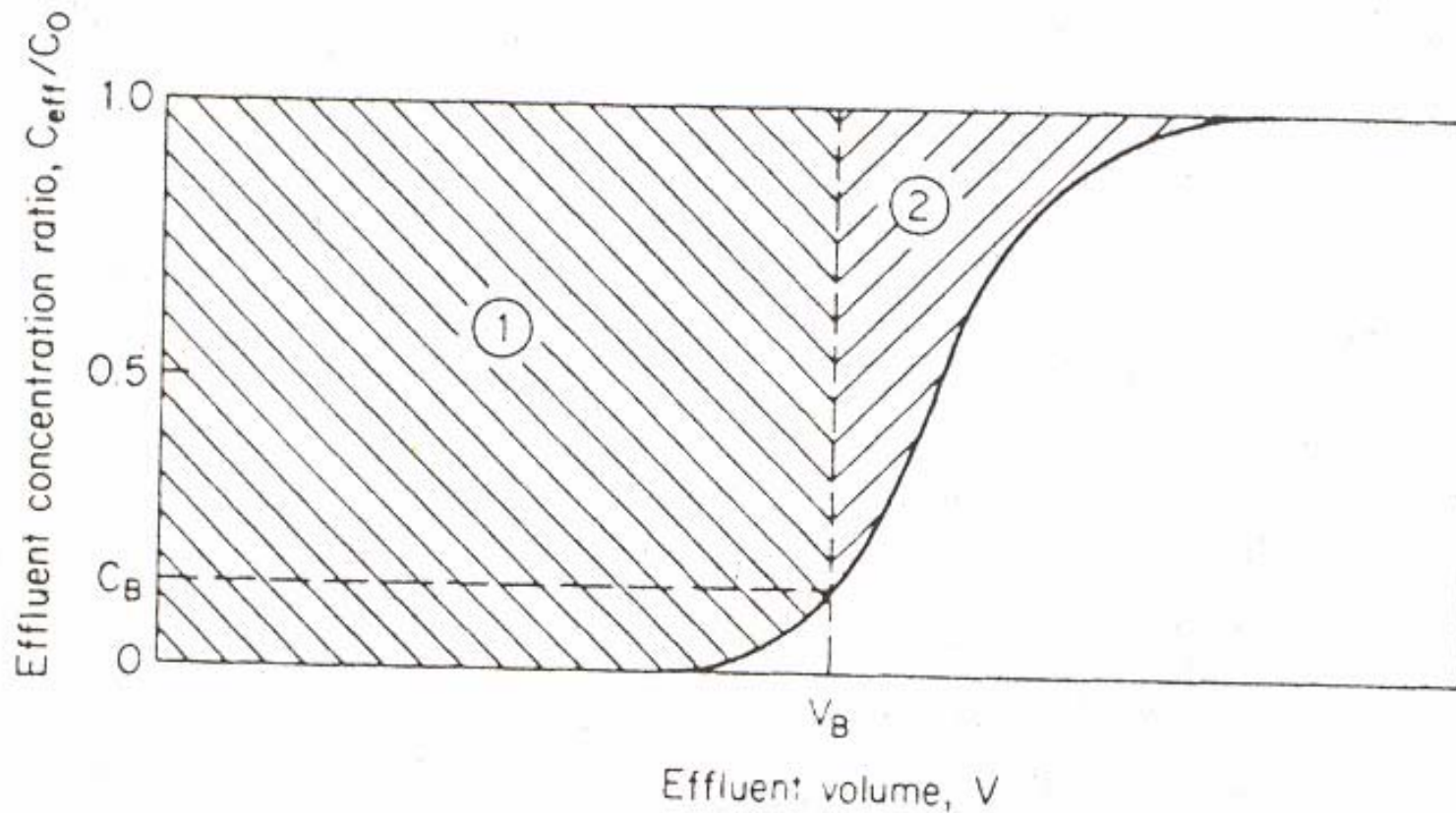
$$\int_{V=0}^V (C_0 - C) dV$$

$$V = 0$$

$$\left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} m \\ \downarrow \\ x \Rightarrow x/m \Rightarrow q \end{array}$$

$$V \Rightarrow V_B \text{ or } V_E$$

# Integration of BTC



Breakthrough Capacity  $\Rightarrow ① \times C_0$

Exhaustion Capacity  $\Rightarrow ① + ② \times C_0$

Capacity Utilization (%) @ Breakthrough  $\Rightarrow ① / (① + ②) \times 100$

Stoichiometric Breakthrough @  $C/C_0 = 0.5$



# Adsorbent Utilization Rate (AUR)

- AUR (mass/volume)

mass of adsorbent in column

- $AUR = \frac{\text{mass of adsorbent in column}}{\text{volume treated to breakthrough, } V_B}$

- From BTC:

$$AUR = \rho_{\text{adsorbent}} (\text{g/L}) / \text{BVs to breakthrough}$$

- From Isotherm:

$$AUR (\text{g/L}) = (C_0) / q_0$$

□  $\text{g/L} = \text{mg/L} \div \text{mg/g}$



# Adsorbent Bedlife (BVs)

- From BTC:

- BVs @ breakthrough

- From Isotherm:

$$\text{Bed-Life} = q_0 / C_0 \times \rho_{\text{adsorbent}}$$

- Bed-Life  $\Rightarrow$  BVs
  - $q_0 \Rightarrow \text{mg/g}$
  - $C_0 \Rightarrow \text{mg/L}$
  - $\rho_{\text{adsorbent}} \Rightarrow \text{apparent density of adsorbent (g/L)}$
  - dimensionless =  $\text{mg/g} \div (\text{mg/L}) \times \text{g/L}$

# Summary

- Isotherms provide an equilibrium estimate of capacity
- Isotherms provide an tool for rapid screening of candidate adsorbents and interferants
- Isotherms provide insight into column capacity

[gamy@spot.colorado.edu](mailto:gamy@spot.colorado.edu)

